The Measurement Problem for Emergent Spacetime in Loop Quantum Gravity

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1 Introduction

It is widely believed that a quantum theory of gravity will not take spacetime to be fundamental, but that our familiar spacetime structure will rather emerge from something more fundamental in a suitable limit of the theory. Any putative quantum theory of gravity must address the quantum measurement problem, and one promising strategy for addressing that problem involves a relational view of states in quantum theory.

Several different ways of implementing this strategy have now been proposed. These include Rovelli's (1996) relational interpretation, QBism (Fuchs 2010: Fuchs, Mermin, Schack 2014), and a pragmatist approach (Healey 2012, 2017*)*. No one has done more to solve the conceptual as well as technical problems posed by a projected quantum theory of gravity than Carlo Rovelli. Anyone attempting to implement a relational strategy toward the measurement problem in quantum gravity has much to learn from Rovelli's path-breaking work in applying his relational interpretation to covariant loop quantum gravity. So what follows should be viewed as an attempt to learn what I can from a critical but deeply sympathetic assessment of that work.

The paper proceeds as follows. Section 2 examines how Rovelli's relational quantum mechanics applies to theories with a fixed background spacetime. This restriction is lifted in section 3, introducing loop quantum gravity—Rovelli's own favored approach to a quantum treatment of the gravitational field. The measurement problem is first discussed in section 4, while section 5 further investigates the application of Rovelli's relational view of quantum theory to loop quantum gravity. In section 6 all these pieces are put together to critically assess Rovelli's use of his relational interpretation to address the measurement problem for loop quantum gravity. The final section 7 draws the morals of this story for alternative relational approaches to quantum theory when applied to the emergence of spacetime from a putative quantum theory of gravity.

2 Rovelli's Relational Quantum Mechanics

In his 1996 paper, Rovelli expressed the key idea behind his relational quantum mechanics as follows (italics in the original):

> "*Quantum mechanics is a theory about the physical description of physical systems relative to other systems, and this is a complete description of the world.*" (p. 1650)

He makes his view clear that what the physical description is relative to is itself simply some other physical system:

> "Observers are not 'physically special systems' in any sense." (p. 1654)

Just as "the observer" to which velocities must be relativized in Galilean relativity may be any physical object (such as a table lamp), so also "the observer" to which the state of a physical system must be relativized in quantum mechanics may be any physical system (such as an electron).

To understand what Rovelli's key idea comes to, we need to ask how quantum systems are to be described physically, and what are the physical systems.

In physics, a system is usually described by assigning it a state: in quantum theory one expects this to be a quantum state, represented mathematically. But just how, and how completely, this mathematical representation describes the system's physical properties is not transparent—especially when the description is taken to be relative to other systems. I address Rovelli's account of the descriptive role of quantum states in the following subsection.

2.1 What is a State?

In ordinary quantum mechanics, the state of a system is represented by a vector in or density operator on Hilbert space. It is common to attribute a determinate value q_i to quantity q on a system if application of the Born rule to that state yields probability 1 that observation of q would reveal value q_i: the eigenstate-eigenvalue link strengthens that conditional to a biconditional by adding that this is the *only* circumstance in which a quantity has a determinate value.

Rovelli (1996, 1997) endorses the eigenstate-eigenvalue link, though only for states relativized to a fixed "observer" system. But he also proposes to replace this notion of the state of a system by the notion of the information that a system has about another system.

Given three physical systems S, O, P, by

"...'q has a value [on S] relative to O', we mean 'relative to P, there is a certain correlation in the S and O states', or, equivalently, 'O has information about q'*."* (Rovelli 1997, p.10) But *"...there is a key irreducible distinction between* P's knowledge that O has information about q and O's knowledge of q*."* (*ibid.*) In this situation does O *have* knowledge of (the value of) q?

If the term ‘knowledge’ has its normal sense, then this question has an easy answer (“No”) in a case where O is a simple quantum system such as an electron that has no capacity for knowledge. But one would expect it typically to have the opposite answer (“Yes”) when O is an alert, competent, human experimentalist. Indeed, *denying* that a human observer can know the value of q leads to a form of skepticism that removes much if not all evidential support for quantum mechanics.

However, Rovelli cannot consistently allow that such knowledge contributes to a complete description of the state of the world.

> “...the incorrect notion at the source of our unease with quantum theory...is the notion of true, universal, observer-independent description of the state of the world.*”* (p. 1637)

So the value of a quantity on S must be relativized to O, and if O is a human observer then his or her knowledge of its value cannot contribute to our knowledge of the observer-independent state of the world!

> “...when we say that a physical quantity takes the value v, we should always (explicitly or implicitly) qualify this statement as: the physical quantity takes the value v with respect to the so and so observer.*”* (p. 1648)

2.2 Systems, Interactions and Processes: Quantum Mechanics

For Rovelli, even relational descriptions are not always available:

> *“*A quantum description of the state of a system S exists only if some system O (considered as an observer) is actually "describing" S, or, more precisely, has interacted with S.*”* (p. 1648)

Systems in quantum mechanics are particles or compounds of them: the distinction between simple and compound is made contextually. The state of a system is represented in its Hilbert space, on which is defined a Hamiltonian determining its undisturbed (Schrödinger picture) evolution.

The occurrence of an interaction between two systems is represented by a non-zero interaction Hamiltonian on their tensor product Hilbert space: This may be arranged to have a specific form in a von Neumann "measurement" that effects a correlation between the initial state of a system S and the final state of another system O: I exhibit this form in section 5.

So in quantum mechanics the history of a system consists of processes of undisturbed evolution punctuated by episodes of interaction. It is represented mathematically by a quantum state that evolves unitarily (in the Schrödinger picture), though only between interactions with other systems is this state assigned to that system alone. For Rovelli, all states are defined relationally, because a state assignment to *any* system is relative to an observer system O that has interacted with it.

2.3 Systems, Interactions and Processes: QFT

In Lagrangian quantum field theory on a fixed background spacetime, systems are quantum fields such as the photon, electron and Higgs fields. Given a decomposition into positive and negative frequencies, the state of a field may be represented in a Fock space: its free evolution is represented by a continuous family of unitary operators on this space. Particles and classical fields may then also be counted as (emergent) systems corresponding to excited states in the associated Fock space.

The occurrence of an interaction between two quantum fields is represented in the total Lagrangian (density) by a non-zero interaction term coupling them. A function of a *single* field operator in the total Lagrangian can imply non-linearity of its resulting Euler-Lagrange equation, indicating a *self-interaction* of that field (as in the gluon field in QCD). To either kind of interaction term corresponds an interaction Hamiltonian whose non-zero value signals the occurrence of an interaction, just as in quantum mechanics.

The paucity of fundamental interactions[1] leaves no freedom to engineer episodic von Neumann "measurements" at will by designing novel fundamental fields. If measurement is to be modeled as a unitary process within quantum theory, this cannot be at the level of fundamental fields. But one may perhaps model von Neumann measurement interactions between *effective* quantum fields that emerge from more fundamental fields in an appropriate limit.

3 Loop Quantum Gravity

In loop quantum gravity there is no background spacetime. So, unlike the photon or electron field, the quantum gravitational field cannot be modeled by an assignment of operators to points or regions of spacetime. Moreover, in pure loop quantum gravity there are no other fermion or boson fields with which the gravitation field might interact.

Since the non-linearity of the Einstein field equations of general relativity at least intimates non-linearity of field equations for a quantized gravitational field, one might think of this field as self-interacting. But what could it be for two systems to interact in loop quantum gravity?

Rovelli and Vidotto (2015, p. 52) say this about Rovelli's (1996) relational quantum mechanics (where loop quantum gravity is not mentioned)

> "...a process is what happens between interactions."

But they say this relational interpretation may be used in the context of quantum gravity, where

> "...a process is not *in* a spacetime region: a process *is* a spacetime region." (*ibid*)

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[1] The Standard Model Lagrangian includes just fundamental strong and electroweak interactions. That leaves gravity as the only other fundamental interaction.

3.1 Systems, States and Interactions in Loop Quantum Gravity

We saw how Rovelli handled the notion of a state in his relational quantum mechanics. How does this work in loop quantum gravity (LQG)? According to section §2.4.3 of (Rovelli and Vidotto, 2015), the relational interpretation of Rovelli, 1996 can be used in the context of quantum gravity. But that section leaves it unclear how to apply this relational interpretation to LQG. We are told that

> "States are descriptions of ways a system can affect another system." (Rovelli and Vidotto 2015, p.52)
>
> "A state is not somewhere in space: it is the description of the way two processes interact, or two spacetime regions passing information to one another." (*ibid*)

It seems that the systems that interact in LQG are spacetime regions, themselves thought of as processes.

> "A spacetime region *is* a process: a state is what happens at its boundary." (*op. cit.,* p. 53)

However, the boundaries marking the division between processes are not fixed, but arbitrarily drawn in spacetime:

> "The physical theory is therefore a description of how arbitrary partitions of nature affect one another."(*ibid*)

This is possible because

> "As noticed, a remarkable aspect of quantum theory is that the boundary between processes can be moved at will. Final total amplitudes are not affected by displacing the boundary between "observed system" and "observing system". The same is true for spacetime: boundaries are arbitrarily drawn in spacetime."(*ibid*)

Total amplitudes are related to processes because the process that is a spacetime region

> is actually a Feynman sum of everything that can happen between its boundaries (*op. cit.,* p. 52)

The total amplitude $\langle W|\Psi\rangle$ associated with the (quantum) state Ψ of the gravitational field on the boundary $\Sigma = \partial M$ of a spacetime region M may be expressed as a Feynman path integral of the field in M (*op. cit.,* pp. 50-1). Assuming this is what the authors are referring to by the phrase 'Feynman sum', we should be able to conclude that the process that is a spacetime region M is simply this total amplitude $W = \langle W|\Psi\rangle$.

But now we have a problem, since an amplitude is a complex number, while a spacetime region is presumably not any kind of number but something in the physical world (perhaps a physical object, perhaps a system of physical relations). There is a further problem understanding the relation between W and a spacetime region at this preliminary stage of the exposition, insofar as

> The explicit construction of W is the main objective in this book. (*op. cit.,* pp. 51-2).

The picture painted in §2.4.3 of Rovelli and Vidotto, 2015 is not of processes undergone by systems during an interval of time while they evolve undisturbed, punctuated by episodes during which they interact with one another. The notion of a background time has disappeared along with that of a background spacetime. That picture has been replaced in LQG by one of systems as processes *constituting* spacetime regions, and their interactions as occurring at the boundaries between these regions.

Perhaps these problems arise because we have been taking this picture too seriously. In LQG, (classical) spacetime regions are supposed to emerge only in a suitable limit of the theory, while at the fundamental level

> "as far as we know today, all that exist in nature are general-covariant quantum fields." (*op. cit.,* p. 19)

Until the construction of W is successfully completed it is impossible to say just what a spacetime region is, making it problematic even to talk of spacetime regions while addressing the measurement problem in loop quantum gravity. Now if spacetime is to emerge only in some limit of the theory one may not expect to be able to talk significantly about spacetime regions away from that limit. So perhaps charity would recommend that the discussion of Rovelli's relational interpretation in section 2.4.3 of this book be taken metaphorically rather than literally. But doing so would leave it unclear how to apply Rovelli's relational interpretation in this context.

Rovelli and Vidotto (2015) give the physical theory of quantum gravity only in chapter 7, where its relational interpretation is not discussed. But I think a brief examination of that theory helps resolve some of the unclarity of the earlier discussion in §2.4.3. The final expression for the transition amplitude W given there (by equations (7.46), (7.47) on p. 141) is stated to give the theory of loop quantum gravity *on a given 2-complex* (Rovelli and Vidotto, p. 142).

A 2-complex is neither a region of spacetime nor a collection of such regions. It is the dual of a bulk triangulation Δ of a *mathematical* four-dimensional space with the topology of (a compact region of) $\mathbb{R}^4$. It is true that Δ is introduced (p. 131) as a triangulation of a compact region M of spacetime. But it is common practice for theoretical physicists to elide the distinction between (regions of) physical spacetime and the elements of a mathematical object used to represent them. And to take M to represent a region of physical spacetime would conflict with LQGs overarching goal of retrieving spacetime only in an appropriate limit of the theory.

Indeed, in section 7.2.3 Rovelli and Vidotto (2015) warn their reader against taking the geometrical picture accompanying their mathematics too literally

> “...the geometrical picture of "tetrahedra," "triangles," etc., must be taken as something meaningful only in some classical approximation and not at the fundamental scale”
> “There are quantum states, formed by quanta of the gravitational field, which have the property of giving rise to something that we describe as a three-dimensional Riemannian geometry in the limit of large quantum numbers.” (p. 140)

They even warn against a naive physical interpretation of such talk

> “...the quanta of space of loop quantum gravity should not be taken too naively as actual entities, but rather as modes of interaction.” (p. 141)

The picture painted in section §2.4.3 of Rovelli and Vidotto (2015) is not of physical processes in spacetime, but of elements of mathematical structure basic to the theory of LQG that eventually predicts transition amplitudes for physical processes that have yet to be interpreted. Interpreting LQG *is* interpreting these processes. Nothing that is said in §2.4.3 of Rovelli and Vidotto (2015) contributes directly to that project. But this discussion may yet contribute *indirectly* to a resolution of the measurement problem in LQG.

4 The Measurement Problem

Leggett (2005) stated the quantum measurement problem this way:

> “most interpretations of quantum mechanics at the microscopic level do not allow definite outcomes to be realized, whereas at the level of our human consciousness it seems a matter of direct experience that such outcomes occur.” (p. 871)

Rovelli responds by taking a measurement interaction between S and O to have an outcome relative to O, but not relative to an external O' before it interacts with them, as in this picture by Rovelli (2004).

Figure 5.1: The observer observed.

This depicts O and O' as *human* observer systems, despite Rovelli's insistence that observers are not physically special systems in any sense. But while O must be a physically special system to be capable of conscious human *experience* of the outcome relative to O, Rovelli can consistently claim that *there is* such an outcome relative to O whether or not O is physically special in this way. In his view an outcome relative to an observer need not be experienced by that observer. Insofar as typical observer systems are not conscious, most outcomes of measurement interactions are not experienced.

What should we say about O' prior to any interaction with S or O? Certainly O' can not then consciously experience an outcome, whether or not it has the capacity for conscious human experience. Rovelli goes further by denying that *there is* then any outcome relative to O': For there to be any outcome relative to O', O' must first interact appropriately with S or O. So before O' interacts with S or O, there is no outcome relative to O': there is only an outcome relative to O.

Despite the deliverances of conscious human experience, quantum measurements have no objective, observer-independent outcomes in Rovelli's relational quantum mechanics.

5 Relational Loop Quantum Gravity

Rovelli's (1996) exposition of relational quantum mechanics leans heavily on the model of simple von Neumann interactions between quantum systems such as S, O that correlate an initial pure eigenstate $|q_i\rangle_S$ of $\hat{q}$ on S with a corresponding final pure eigenstate on O.

$$|q_i\rangle_S \otimes |ready\rangle_O \Longrightarrow |q_i\rangle_S \otimes |\text{“}q = q_i\text{”}\rangle_O \tag{1}$$

While "observer system" O need not be physically special for this interaction to yield a value q_i of the quantity q on S relative to O (which may even be an electron), only if O is a special physical system can O be said to have knowledge of q as a result of this interaction. Human scientists are special physical systems that may be idealized as quantum systems: perhaps registration by a non-conscious measuring apparatus or another kind of IGUS (Information-Gathering and Utilizing System) could also be said to constitute knowledge.

How can this model be transposed into the context of loop quantum gravity? The arrow $\Longrightarrow$ of equation (1) cannot be replicated at a fundamental level, since there is no time at a fundamental level in loop quantum gravity. Nor is it clear how to replicate the systems represented in equation (1) if all that exist in nature are general covariant fields.

Suppose we were to take the picture of §2.4.3 of Rovelli and Vidotto (2015) to provide the basis for a fundamental LQG model of a measurement interaction. In that picture an interaction is what happens at the boundary between two spacetime regions. These regions are now the systems that interact, conceived as processes rather than enduring objects. But how are we to model an observer system capable of registering if not experiencing the (relative) outcome of the interaction?

An observer system O may be reasonably idealized in quantum mechanics as an enduring quantum system composed of a vast number of particles with bulk degrees of freedom corresponding to recording states. No analogous idealization is apparent in loop quantum gravity: an observer system is not reasonably idealized as a spacetime region! But eventually, fermion and boson quantum fields must be integrated into loop quantum gravity. So one might hope to use these to construct an idealized model of an observer system in LQG. However this remains a distant aspiration. There is no present prospect of a satisfactory model of a measurement interaction at the fundamental level in LQG.

But we saw in section 3.1 that the discussion of §2.4.3 of Rovelli and Vidotto (2015) is best taken to describe a geometrical picture not of the structure of (processes in) spacetime in LQG, but of the mathematics of transition amplitudes defined over bulk triangulations and their duals in $\mathbb{R}^4$. It is a category mistake to identify these mathematical structures on $\mathbb{R}^4$ with regions of physical spacetime. LQG does not itself *describe* regions of spacetime, either classical or nonclassical. It provides (picturable) mathematical models that may be used to calculate transition amplitudes. The problem is to understand what these are transition amplitudes *for* in the absence of objects or processes in physical spacetime.

Now it is a constraint on the empirical adequacy of LQG that classical spacetime emerge in a suitable limit of the theory. Assuming that constraint is met, physical spacetime may be said to exist in that limit, along with whatever physical processes occur within it. At a fundamental level, those processes may be modeled by theories of fermion and boson quantum fields as well as the LQG field. It is a constraint on the empirical adequacy of those other field theories that ordinary quantum mechanics emerge in a suitable limit, just as it is a constraint on the empirical adequacy of LQG that gravitational processes in emergent classical spacetime be correctly modeled. So while the von Neumann model (1) cannot be replicated at the fundamental level in LQG, it must be recoverable from that theory in a suitable limit in which systems like S, O emerge as objects that endure in classical spacetime. In that limit, the transition amplitudes for gravitational processes calculated from LQG may be applied in models like (1) since such a unitary transformation of states is calculable from them.

While LQG cannot model von Neumann interactions at a fundamental level, its transition amplitudes must be compatible with the existence of such interactions in a suitable limit in which classical spacetime emerges, along with enduring physical systems to play the roles of system S and observer O. That is how LQG may be given a relational interpretation along the lines of Rovelli, 1996. But does this help solve the measurement problem for LQG?

6 The Measurement Problem in Relational LQG

There can be no *absolute* definite outcomes in relational loop quantum gravity, since all states are relative to systems. The measurement problem is then to reconcile merely *relative* outcomes with the fact that at the level of our human consciousness it seems a matter of direct experience that definite outcomes occur. In relational quantum mechanics, this may be possible if one models a human observer as a (special kind of) enduring physical system.

Even if successful, this strategy comes at a steep cost, because it sacrifices an ideal of scientific objectivity: It has always been a basic scientific norm that a human observer's sincere report states what has objectively happened. This norm is violated if O's sincere report of the outcome of his quantum measurement merely reports its outcome *relative to him*, while for P the measurement then *has* no outcome, but only acquires an outcome *relative to her* after she has (directly or indirectly) interacted with O or S.

Perhaps this norm is not inviolable. Perhaps science could continue without it as long as no further observations, by O, P or any other observer, could make this violation manifest? Rovelli (1996,1997) has appealed to quantum models of measurement-type interactions to show why it must remain hidden: In these models, repeated careful measurements by any observer, on the original system or on other observers, will yield *relative* outcomes in *apparent* conformity to that norm.

The absence of a LQG model of a human observer presents an obstacle to implementing such a relational solution to the measurement problem in loop quantum gravity. A human observer is a very special physical system for which we have no model in loop quantum gravity: There is no present prospect of modeling even a simple recording apparatus in LQG. But as we saw in §5 it is not necessary to model a human observer directly in LQG in order to implement a relational interpretation of that theory. Within an empirically adequate LQG, a model of a measurement interaction in ordinary quantum mechanics in a classical background spacetime will provide a platform on which to build such a relational interpretation. Whether such a platform is available hinges on the successful treatment of the limiting process in LQG.

7 Morals of the Story

We have learned two morals from this exploration of Rovelli's strategy for solving the measurement problem in loop quantum gravity.

1. Relativizing values of magnitudes as well as quantum states to systems (and only after they have interacted) not only abandons the notion of a true, universal, observer-independent description of the state of the world, but also threatens a basic norm of scientific objectivity.

2. In addressing the quantum measurement problem, it is neither necessary nor possible to model an observer system at a fundamental level within LQG. A physical characterization in terms of structures (including classical spacetime) that emerge from LQG only in an appropriate limit is all that is needed. So a resolution to the measurement problem is not a prerequisite for the recovery of spacetime in quantum gravity.[2]

Taking these morals to heart suggests certain modifications to Rovelli's relational strategy:

By allowing relativization of quantum states *but not values of magnitudes* to physical systems it may be possible to retain this basic norm of scientific objectivity, and even the notion of a true, universal, observer-independent description of the state of the world. This means rejecting the eigenvalue-eigenstate link. It is not necessary to treat all physical systems on a par when relativizing *quantum* states to physical systems. Relativization to physically special observer systems will suffice. A quantum theory of gravity is not itself required to say what makes these observer systems special in order effectively to address the measurement problem: That characterization may be given in terms of physical structures (including spacetime) that emerge only in a suitable limit of the theory.

A pragmatist approach to quantum theory (Healey 2012, 2017) takes these morals to heart when applied to the emergence of spacetime in a prospective quantum theory of gravity (Healey, forthcoming).

References

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[2] But of course an adequate understanding of *any* quantum theory does require that this problem be addressed and solved or dissolved at some stage. Note that in drawing this moral I register disagreement with Wüthrich (2017, 332) while gratefully acknowledging the stimulus his paper and subsquent comments have provided to my own thinking.

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